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THE THEORY OF ORTHOTROPIC VISCOELASTIC SHEAR DEFORMABLE COMPOSITE FLAT PANELS AND THEIR DYNAMIC STABILITY

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Abstract—This paper deals with an exact approach to the dynamic stability of orthotropic shear-deformable viscoelastic flat plates subjected to in-plane uni/biaxial edge load systems. In deriving the associated governing equations a Boltzmann hereditary law is used and in addition transverse shear deformation, transverse normal stress and rotatory inertia effects are incorporated. The integro-differential equations governing the stability of simply-supported flat plates are solved in the Laplace transform (LT) space in order to determine the critical in-plane edge loads yielding the asymptotic instability of flat plates. The stability analysis allows one to obtain the nature of the loss of stability, i.e. either by divergence or by flutter. Numerical applications are presented and pertinent conclusions are formulated.

INTRODUCTION

Tremendous interest in the analysis of fiber-reinforced composite plate (and shell) like structures has been manifested in the last few years in the field literature. This interest is due to the advent and increased use of high modulus, high strength, low weight composite materials in the various fields of modern technology. Among the multitude of applications they include, e.g. the high-speed aircraft and aerospace structures, rocket engines, turbine blades, etc. Due to the high temperature gradients experienced by these structures, their constituent materials exhibit a time-dependent behavior which could be modelled by a linear (or nonlinear) constitutive law. In addition, due to their weak rigidity in the transverse shear direction, the theory of flat (or curved) panels composed of composite materials, requires the incorporation of transverse shear deformation effects. In spite of its evident importance, the research in this field appears to be somewhat scarce. In their monograph, Malmeister *et al.* (1980) peformed a stability analysis for the transversely isotropic viscoelastic panels undergoing cylindrical bending. However, in their approach, the viscoelastic properties are considered in the transverse shear direction only. Since the extensional moduli in the direction perpendicular to the fibers also exhibit a time-dependent behavior (due to the presence of the matrix) this constitutes an arguable restriction imposed on the material behavior. Wilson and Vinson (1984) analyzed the stability of rectangular, viscoelastic orthotropic plates subject to biaxial compression. In their analysis the equations governing the stability are obtained by using a quasi-elastic approximation which overlooks the hereditary material behavior. Sims (1972) performed a similar quasi-elastic analysis of the problem, thereby implying an instantaneous time-dependent behavior as opposed to the hereditary one.

In this study linear viscoelasticity theory is used to analyze the dynamic stability of composite, viscoelastic flat plates subjected to in-plane, uni/biaxial edge loads. To this end an exact dynamic approach has been used. In deriving the associated governing equations, a three-dimensional linearly viscoelastic, hereditary constitutive law is assumed. In addition, having in view that composite-type structures exhibit weak rigidities in transverse shear,

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the associated governing equations account for the transverse shear deformations, as well as for the transverse normal stress effect, which has hitherto been neglected. The integro-differential equations governing the stability are derived using the elastic–viscoelastic correspondence principle applied to the equations derived within the elastic range in Librescu and Reddy (1986). The governing equations are solved for simply-supported boundary conditions by using the Laplace transform technique, thus yielding the characteristic equation of the system.

In order to predict the effective time-dependent properties of the orthotropic plate, an elastic behavior is assumed for the fiber, whereas the matrix is considered as linearly viscoelastic. In this connection, towards the goal of evaluating the nine independent properties of the orthotropic viscoelastic material in terms of its isotropic constituents, the micromechanical relations developed in Aboudi (1987) are considered in conjunction with the correspondence principle of linear viscoelasticity.

The stability behavior analyzed here concerns the determination of the critical in-plane normal edge loads yielding the asymptotic instability of the plate. The problem is studied as an eigenvalue problem.

The general dynamic instability solutions are compared with their quasi-static counterparts. Comparisons of the various solutions obtained in the framework of the third-order transverse shear deformation theory (TSDT) are made with its first-order counterpart (FSDT). Several special cases are considered and pertinent numerical results are compared with the very few available in the field literature. Comparisons between the TSDT, FSDT and the classical Kirchhoff theory of plates are also presented.

PRELIMINARIES

The case of a flat plate of uniform thickness h is considered. By $S_\pm$ we denote the upper and lower bounding planes of the plate (defined by $x_3 = \pm h/2$) symmetrically located with respect to its mid-plane σ (defined by $x_3 = 0$), while by Ω we denote the edge boundary surface.

The points of the three-dimensional space of the plate will be referred to a rectangular Cartesian system of coordinates x_i, where x_α ($\alpha = 1, 2$) denote the in-plane coordinates, x_3 being the coordinate normal to the plane $x_3 = 0$. Throughout the analysis, unless otherwise stated, the Einsteinian summation convention is employed where Greek indices range from 1 to 2, while Latin indices range from 1 to 3.

BASIC EQUATIONS

Strain–displacement equations

For a third-order bending theory (TSDT) of plates which retains the assumption of inextensibility of the transverse normal elements, the following representation of the displacement field across the thickness of the plate was postulated (Librescu and Reddy, 1986):

$$V_\alpha = x_3 \overset{(1)}{V}_\alpha + (x_3)^3 \overset{(3)}{V}_\alpha$$

$$V_3 = \overset{(0)}{V}_3 \tag{1}$$

where

$$V_i = V_i[x_\alpha, x_3, t] \quad \text{while} \quad \overset{(n)}{V}_i = \overset{(n)}{V}_i[x_\alpha, t].$$

At this stage we note that the above representation of V_α allows one to fulfill the static boundary conditions on the external bounding planes $S_\pm$ implying the absence of tangential external loads. The linear strain tensor is written as

$$e_{ij}[x_\alpha, x_3, t] = \tfrac{1}{2}(V_{i,j}+V_{j,i}). \tag{2}$$

Upon substituting eqns (1) into eqn (2), we obtain

$$\begin{aligned} 2e_{\alpha\beta} &= x_3(\overset{(1)}{V}_{\alpha,\beta}+\overset{(1)}{V}_{\beta,\alpha})+(x_3)^3(\overset{(3)}{V}_{\alpha,\beta}+\overset{(3)}{V}_{\beta,\alpha}) \\ 2e_{\alpha 3} &= \overset{(1)}{V}_\alpha+3(x_3)^2\overset{(3)}{V}_\alpha+\overset{(0)}{V}_{3,\alpha} \\ e_{33} &= 0 \end{aligned} \tag{3}$$

where $e_{\alpha\beta}$, $e_{\alpha 3}$ and e_{33} denote the in-plane, transverse shear and transverse normal strain components, respectively. By virtue of the assumption implying the absence of distributed external moments $\overset{(1)}{p}_\alpha$, i.e.

$$[\sigma_{\alpha 3}x_3]_{-h/2}^{+h/2} = \overset{(1)}{p}_\alpha \to 0 \tag{4}$$

it may be shown (see for more details Librescu and Reddy (1986)) that the following relationship holds:

$$\overset{(3)}{V}_\lambda = -\frac{4}{3h^2}(\overset{(0)}{V}_{3,\lambda}+\overset{(1)}{V}_\lambda). \tag{5}$$

Equation (5) in conjunction with eqns (1) reveals that the bending theory of plates may be reduced to the determination of only three displacement components (i.e. $\overset{(1)}{V}_\lambda$ and $\overset{(0)}{V}_3$).

Constitutive equations

As a result of the principle of superposition of the linear viscoelasticity theory we may obtain Boltzmann's hereditary constitutive law (see, e.g. Pipkin (1972) and Christensen (1982)). For a three-dimensional linearly viscoelastic anisotropic material this may be written as (Malmeister *et al.*, 1980)

$$\sigma_{ij}[t] = e_{mn}[t]E_{ijmn}[0]+\int_0^t \dot{E}_{ijmn}[\tau]e_{mn}[t-\tau]\,\mathrm{d}\tau \tag{6}$$

where $E_{ijmn}[t]$ are the relaxation moduli; t and τ denote the current and delayed time variables, respectively. In eqn (6) the first term corresponds to the elastic material behavior. The Laplace transform (LT) of eqn (6) yields

$$\bar{\sigma}_{ij} = s\bar{E}_{ijmn}\bar{e}_{mn} \tag{7}$$

where the overbars denote the LT with s as the LT variable, while the overdots denote time derivatives. Considering the elastic part of constitutive law (6), we may express it in the following convenient form (Librescu, 1975):

$$\begin{aligned} \sigma_{\alpha\beta} &= \tilde{E}_{\alpha\beta\omega\pi}e_{\omega\pi}+\delta_\Lambda\hat{E}_{\alpha\beta 33}\sigma_{33} \\ \sigma_{\omega 3} &= 2E_{\omega 3\lambda 3}e_{\lambda 3} \end{aligned} \tag{8}$$

where $\sigma_{\alpha\beta}$ and $\sigma_{\omega 3}$ denote in-plane and transverse shear-stress components, while

$$\begin{aligned} \tilde{E}_{\alpha\beta\omega\pi} &= E_{\alpha\beta\omega\pi}-\frac{E_{\alpha\beta 33}E_{33\omega\pi}}{E_{3333}} \\ \hat{E}_{\alpha\beta 33} &= \frac{E_{\alpha\beta 33}}{E_{3333}} \end{aligned} \tag{9}$$

denote the reduced elastic moduli, δ_A being a tracer identifying the presence of σ_{33} (which takes the value 0 or 1, according to whether this influence is ignored or included).

Equations of motion

The equations of motion for a three-dimensional linear continuum are as follows:

$$\sigma_{\alpha i,i} + \rho H_\alpha = \rho \ddot{V}_\alpha \tag{10a}$$

$$\sigma_{3\beta,\beta} + \sigma_{33,3} + \rho H_3 = \rho \ddot{V}_3 \tag{10b}$$

where ρ denotes the mass density of the medium, and H_i the body forces per unit mass.

In order to formulate the refined bending theory of plates in terms of the basic unknowns $\overset{(1)}{V}_\alpha$ and $\overset{(0)}{V}_3$, we need three macroscopic equations of motion. These are obtained by considering the moment of order one of the first two equations of motion, eqn (10a), and the zeroth-order moment of the third equation of motion, eqn (10b). In the absence of body forces, these two-dimensional equations of motion read

$$\overset{(1)}{L}_{\alpha\beta,\beta} - \overset{(0)}{L}_{\alpha 3} = \delta_C \overset{(1)}{f}_\alpha$$

$$\overset{(0)}{L}_{\alpha 3,\alpha} + \overset{(0)}{p}_3 - \delta_D \rho h \overset{(\ddot{0})}{V}_3 = 0 \tag{11}$$

where

$$\overset{(1)}{L}_{\alpha\beta}[x_1, x_2, t]\left(\equiv \int_{-h/2}^{h/2} \sigma_{\alpha\beta} x_3 \, \mathrm{d}x_3\right)$$

$$\overset{(0)}{L}_{\alpha 3}[x_1, x_2, t]\left(\equiv \int_{-h/2}^{h/2} \sigma_{\alpha 3} \, \mathrm{d}x_3\right) \tag{12}$$

define the moment resultants and the transverse shear resultant, respectively, while

$$\overset{(0)}{p}_3[x_1, x_2, t]\ (\equiv (\sigma_{33})|^{h/2}_{-h/2}) \tag{13}$$

define the distributed external transverse loads (per unit area), while ρ denotes the mass density of the medium. In eqns (11) δ_D and δ_C are two tracers identifying the effect of the transverse inertia and rotary inertia term

$$\overset{(1)}{f}_\alpha\left(\equiv \int_{-h/2}^{h/2} \rho \ddot{V}_\alpha x_3 \, \mathrm{d}x_3\right)$$

respectively. Towards the goal of representing the governing equations in terms of the basic variables $\overset{(1)}{V}_\alpha$ and $\overset{(0)}{V}_3$ eqns (11) are to be used in conjunction with eqns (12), (8), (3), (5) and (10a) (which is integrated across the segment $[0, x_3)$ in order to determine σ_{33}). This yields the following governing equations of elastic orthotropic flat plates (TSDT):

$$\tilde{E}_{\alpha\beta\omega\pi} \overset{(0)}{V}_{3,\omega\pi\beta} - 4\tilde{E}_{\alpha\beta\omega\pi} \overset{(1)}{V}_{\omega,\pi\beta} + 4\delta_A \frac{E_{\alpha\beta 33}}{E_{3333}} E_{\omega 3\lambda 3}(\overset{(1)}{V}_{\lambda,\omega\beta} + \overset{(0)}{V}_{3,\omega\lambda\beta})$$

$$- 5\delta_A \delta_B \frac{E_{\alpha\beta 33}}{E_{3333}} \rho \overset{(\ddot{0})}{V}_{3,\beta} + \frac{40}{h^2} E_{\alpha 3\lambda 3} \overset{(1)}{V}_\lambda + \frac{40}{h^2} E_{\alpha 3\lambda 3} \overset{(0)}{V}_{3,\lambda} + \delta_C \frac{60}{h^3} \overset{(1)}{f}_\alpha = 0$$

$$\tfrac{2}{3} h E_{\omega 3\lambda 3}(\overset{(1)}{V}_{\lambda,\omega} + \overset{(0)}{V}_{3,\lambda\omega}) + \overset{(0)}{p}_3 - \delta_D \rho h \overset{(\ddot{0})}{V}_3 = 0. \tag{14}$$

In these equations

$$\overset{(1)}{f}_\alpha = \frac{\rho h^3}{60}\,(4\overset{(\ddot{1})}{V}_\alpha - \overset{(\ddot{0})}{V}_{3,\alpha}) \tag{15}$$

while $F_{\lambda 3\omega 3}$ denotes the elastic transverse shear compliance when $\lambda = \omega$, and $F_{\lambda 3\omega 3} = 0$ for $\lambda \neq \omega$. Also the tracer δ_B identifies the dynamic effect of σ_{33}.

Within the first-order transverse shear deformation theory (FSDT) the following representation of the displacement field is postulated:

$$\begin{aligned} V_\alpha[x_\omega, x_3, t] &= x_3 \overset{(1)}{V}_\alpha \\ V_3[x_\omega, x_3, t] &= \overset{(0)}{V}_3. \end{aligned} \tag{16}$$

Neglecting the influence of σ_{33} in the constitutive law (i.e. $\delta_A = 0$ in eqns (8)), and using the procedure outlined for the TSDT we obtain the following equation governing the bending of transverse shear deformable orthotropic flat plates (FSDT) (Librescu, 1975):

$$\begin{aligned} &\frac{h^3}{12}\tilde{E}_{\alpha\beta\mu\rho}\overset{(1)}{V}_{\mu,\rho\alpha} - K^2 h E_{\beta 3\lambda 3}(\overset{(1)}{V}_\lambda + \overset{(0)}{V}_{3,\lambda}) - \delta_C m_1 \overset{(\ddot{1})}{V}_\beta = 0 \\ &K^2 h E_{\beta 3\lambda 3}(\overset{(1)}{V}_{\lambda,\beta} + \overset{(0)}{V}_{3,\lambda\beta}) + \overset{(0)}{p}_3 - \delta_D m_0 \overset{(\ddot{0})}{V}_3 = 0 \end{aligned} \tag{17}$$

where K^2 is the transverse shear correction factor, $m_0 = \rho h$ and $m_1 = \dfrac{\rho h^3}{12}$.

THE EQUATIONS GOVERNING THE STABILITY OF VISCOELASTIC ORTHOTROPIC FLAT PLATES

In the absence of distributed external moments (implying $\overset{(1)}{p}_\alpha = 0$), the stability equation of an orthotropic elastic plate may be obtained formally by replacing in the governing equations, $\overset{(0)}{p}_3$ by $\overset{(0)}{p}_3 + \overset{(0)}{L}_{11}\overset{(0)}{V}_{3,11} + \overset{(0)}{L}_{22}\overset{(0)}{V}_{3,22} + 2\overset{(0)}{L}_{12}\overset{(0)}{V}_{3,12}$ (see, e.g. Volmir (1967) and Ambartsumian (1970)), where $\overset{(0)}{L}_{11}$, $\overset{(0)}{L}_{22}$ and $\overset{(0)}{L}_{12}$ play the role of in-plane edge loads. A full deduction of stability equations was carried out in Chandiramani (1987). Furthermore, the viscoelastic counterparts of elastic stability equations can be obtained by taking the LT of the latter ones and then replacing therein the moduli and compliances by their Carson transforms (CT). Employment of the above procedure in eqns (14) yields, in the Laplace transform space, the equations governing the stability of viscoelastic, orthotropic, flat plates. They are

$$\begin{aligned} &\bar{\bar{E}}^*_{\alpha\beta\omega\pi}\overset{(\bar{0})}{V}_{3,\omega\pi\beta} - 4\bar{\bar{E}}^*_{\alpha\beta\omega\pi}\overset{(\bar{1})}{V}_{\omega,\pi\beta} + 4\delta_A\frac{\bar{E}^*_{\alpha\beta 33}}{\bar{E}^*_{3333}}\bar{E}^*_{\omega 3\lambda 3}(\overset{(\bar{1})}{V}_{\lambda,\omega\beta} + \overset{(\bar{0})}{V}_{3,\omega\lambda\beta}) \\ &\quad - 5\delta_A\delta_B\frac{\bar{E}^*_{\alpha\beta 33}}{\bar{E}^*_{3333}}\rho(s^2\overset{(\bar{0})}{V}_{3,\beta} - s\overset{(0)}{V}_{3,\beta}[0] - \overset{(\dot{0})}{V}_{3,\beta}[0]) + \frac{40}{h^2}\bar{E}^*_{\alpha 3\lambda 3}\overset{(\bar{1})}{V}_\lambda \\ &\quad + \frac{40}{h^2}\bar{E}^*_{\alpha 3\lambda 3}\overset{(\bar{0})}{V}_{3,\lambda} + \frac{60}{h^3}\delta_C\overset{(\bar{1})}{\bar{f}}_\alpha = 0 \end{aligned} \tag{18a}$$

$$\begin{aligned} &\tfrac{2}{3}h\bar{E}^*_{\omega 3\lambda 3}(\overset{(\bar{1})}{V}_{\lambda,\omega} + \overset{(\bar{0})}{V}_{3,\lambda\omega}) + \overset{(\bar{0})}{p}_3 + \mathscr{L}\{\overset{(0)}{L}_{11}\overset{(0)}{V}_{3,11} \\ &\quad + \overset{(0)}{L}_{22}\overset{(0)}{V}_{3,22} + 2\overset{(0)}{L}_{12}\overset{(0)}{V}_{3,12}\} - \delta_D\rho h(s^2\overset{(\bar{0})}{V}_3 - s\overset{(0)}{V}_3[0] - \overset{(\dot{0})}{V}_3[0]) = 0 \end{aligned} \tag{18b}$$

where

$$\overline{\overset{(1)}{f}}_\alpha = \rho\left(\frac{h^3}{15}[s^2\overline{\overset{(1)}{V}}_\alpha - s\overset{(1)}{V}_\alpha[0] - \dot{\overset{(1)}{V}}_\alpha[0]] - \frac{h^3}{60}[s^2\overline{\overset{(0)}{V}}_{3,\alpha} - s\overset{(0)}{V}_{3,\alpha}[0] - \dot{\overset{(0)}{V}}_{3,\alpha}[0]]\right). \tag{19}$$

In eqns (18) and (19) the overbar affecting a quantity denotes its LT while an overbar followed by a star, i.e. ($^{-}*$) denotes its CT. Inverting eqns (18) and (19) into the time domain, we obtain

$$\begin{aligned}
&\int_0^t \dot{\tilde{E}}_{\alpha\beta\omega\pi}[t-\tau]\overset{(0)}{V}_{3,\omega\pi\beta}[\tau]\,\mathrm{d}\tau + \tilde{E}_{\alpha\beta\omega\pi}[0]\overset{(0)}{V}_{3,\omega\pi\beta}[t] \\
&-4\int_0^t \dot{\tilde{E}}_{\alpha\beta\omega\pi}[t-\tau]\overset{(1)}{V}_{\omega,\pi\beta}[\tau]\,\mathrm{d}\tau - 4\tilde{E}_{\alpha\beta\omega\pi}[0]\overset{(1)}{V}_{\omega,\pi\beta}[t] \\
&+4\delta_A\int_0^t \dot{\hat{E}}_{\alpha\beta\omega\lambda}[t-\tau][\overset{(0)}{V}_{\lambda,\omega\beta}[\tau] + \overset{(0)}{V}_{3,\omega\lambda\beta}[\tau]]\,\mathrm{d}\tau \\
&+4\delta_A\hat{E}_{\alpha\beta\omega\lambda}[0][\overset{(1)}{V}_{\lambda,\omega\beta}[t] + \overset{(0)}{V}_{3,\omega\lambda\beta}[t]] \\
&-5\delta_A\delta_B\rho\int_0^t \dot{\hat{E}}_{\alpha\beta 33}[t-\tau]\ddot{\overset{(0)}{V}}_{3,\beta}[\tau]\,\mathrm{d}\tau - 5\delta_A\delta_B\rho\hat{E}_{\alpha\beta 33}[0]\ddot{\overset{(0)}{V}}_{3,\beta}[t] \\
&+\frac{40}{h^2}\int_0^t \dot{E}_{\alpha 3\lambda 3}[t-\tau][\overset{(1)}{V}_\lambda[\tau] + \overset{(0)}{V}_{3,\lambda}[\tau]]\,\mathrm{d}\tau \\
&+\frac{40}{h^2}E_{\alpha 3\lambda 3}[0][\overset{(1)}{V}_\lambda[t] + \overset{(0)}{V}_{3,\lambda}[t]] + \frac{60}{h^3}\delta_C\overset{(1)}{f}_\alpha[t] = 0
\end{aligned} \tag{20}$$

$$\begin{aligned}
&\tfrac{2}{3}h\int_0^t \dot{E}_{\omega 3\lambda 3}[t-\tau][\overset{(1)}{V}_{\lambda,\omega}[\tau] + \overset{(0)}{V}_{3,\lambda\omega}[\tau]]\,\mathrm{d}\tau + \tfrac{2}{3}hE_{\omega 3\lambda 3}[0][\overset{(1)}{V}_{\lambda,\omega}[t] + \overset{(0)}{V}_{3,\lambda\omega}[t]] \\
&+\overset{(0)}{p}_3[t] + \overset{(0)}{L}_{11}[t]\overset{(0)}{V}_{3,11}[t] + \overset{(0)}{L}_{22}[t]\overset{(0)}{V}_{3,22}[t] + 2\overset{(0)}{L}_{12}[t]\overset{(0)}{V}_{3,12}[t] - \delta_D\rho h\ddot{\overset{(0)}{V}}_3[t] = 0.
\end{aligned} \tag{21}$$

In eqns (20) and (21) the following notations have been introduced:

$$\begin{aligned}
\tilde{E}_{\alpha\beta\omega\pi}[t] &\equiv \mathscr{L}^{-1}\left\{\frac{1}{s}\left(\bar{E}^*_{\alpha\beta\omega\pi} - \frac{\bar{E}^*_{\alpha\beta 33}\bar{E}^*_{33\omega\pi}}{\bar{E}^*_{3333}}\right)\right\} \\
\hat{E}_{\alpha\beta\omega\lambda}[t] &\equiv \mathscr{L}^{-1}\left(\frac{1}{s}\bar{E}^*_{\omega 3\lambda 3}\frac{\bar{E}^*_{\alpha\beta 33}}{\bar{E}^*_{3333}}\right) \\
\hat{E}_{\alpha\beta 33}[t] &\equiv \mathscr{L}^{-1}\left\{\frac{1}{s}\frac{E_{\alpha\beta 33}}{\bar{E}^*_{3333}}\right\}.
\end{aligned} \tag{22}$$

Equations (20) and (21) represent the system of equations (in LT space) governing the stability of viscoelastic, orthotropic, plates in the framework of the TSDT. By dropping, in the above equations, the terms involving the time derivatives of the material properties, we obtain the elastic counterparts of these equations.

A similar procedure applied to eqns (17) yields the LT of the equations governing the stability of plates within FSDT. These are

$$\frac{h^3}{12}\bar{\bar{E}}^*_{\alpha\beta\mu\rho}\overline{\overset{(1)}{V}}_{\mu,\rho\alpha}-K^2h\bar{E}^*_{\beta3\lambda3}(\overline{\overset{(1)}{V}}_{\lambda}+\overline{\overset{(0)}{V}}_{3,\lambda})-\delta_{C}m_1(s^2\overline{\overset{(1)}{V}}_{\beta}-s\overset{(1)}{V}_{\beta}[0]-\overset{(\dot{1})}{V}_{\beta}[0])=0 \tag{23}$$

and

$$K^2h\bar{E}^*_{\beta3\lambda3}(\overline{\overset{(1)}{V}}_{\lambda,\beta}+\overline{\overset{(0)}{V}}_{3,\lambda\beta})+\overline{\overset{(0)}{p}}_3+\mathscr{L}\{\overset{(0)}{L}_{11}\overline{\overset{(0)}{V}}_{3,11}+\overset{(0)}{L}_{22}\overline{\overset{(0)}{V}}_{3,22}$$
$$+2\overset{(0)}{L}_{12}\overline{\overset{(0)}{V}}_{3,12}\}-\delta_{D}m_0(s^2\overline{\overset{(0)}{V}}_3-s\overset{(0)}{V}_3[0]-\overset{(\dot{0})}{V}_3[0])=0. \tag{24}$$

Inversion of eqns (23) and (24) into the time domain yields

$$\frac{h^3}{12}\int_0^t\dot{\tilde{E}}_{\alpha\beta\mu\rho}[t-\tau]\overset{(1)}{V}_{\mu,\rho\alpha}[\tau]\,\mathrm{d}\tau+\frac{h^3}{12}\tilde{E}_{\alpha\beta\mu\rho}[0]\overset{(1)}{V}_{\mu,\rho\alpha}[t]$$
$$-K^2h\int_0^t\dot{E}_{\beta3\lambda3}[t-\tau][\overset{(1)}{V}_{\lambda}[\tau]+\overset{(0)}{V}_{3,\lambda}[\tau]]\,\mathrm{d}\tau$$
$$-K^2hE_{\beta3\lambda3}[0][\overset{(1)}{V}_{\lambda}[t]+\overset{(0)}{V}_{3,\lambda}[t]]-\delta_{C}m_1\overset{(\ddot{1})}{V}_{\beta}[t]=0 \tag{25}$$

and

$$K^2h\int_0^t\dot{E}_{\beta3\lambda3}[t-\tau][\overset{(1)}{V}_{\lambda,\beta}[\tau]+\overset{(0)}{V}_{3,\lambda\beta}[\tau]]\,\mathrm{d}\tau+K^2hE_{\beta3\lambda3}[0][\overset{(1)}{V}_{\lambda,\beta}[t]+\overset{(0)}{V}_{3,\lambda\beta}[t]]$$
$$+\overset{(0)}{p}_3[t]+\overset{(0)}{L}_{11}[t]\overset{(0)}{V}_{3,11}[t]+\overset{(0)}{L}_{22}[t]\overset{(0)}{V}_{3,22}[t]+2\overset{(0)}{L}_{12}[t]\overset{(0)}{V}_{3,12}[t]-\delta_{D}m_0\overset{(\ddot{0})}{V}_3[t]=0. \tag{26}$$

Equations (20) and (21) and their FSDT counterparts (i.e. eqns (25) and (26)) represent a system of linear integro-differential equations which could be used either in a dynamic response or dynamic stability analysis of the system subjected to time-dependent, bi-axial, in-plane edge loads.

It is also worthwhile to note at this point that for a stability analysis the transverse load $\overset{(0)}{p}_3[t]$, as well as the terms associated with the initial conditions for the displacement field which appear in the governing equations should be dropped (Porter, 1968).

Boundary conditions

Equations (20) and (21) as well as eqns (25) and (26) represent sixth-order governing equation systems the solutions of which must be determined in conjunction with the prescribed boundary conditions (which are three at each edge). For a simply-supported plate (hinged–free in the normal direction), we have the following boundary conditions:

$$\overset{(1)}{V}_2=\overset{(0)}{V}_3=\overset{(1)}{L}_{11}=0 \quad \text{at} \quad x_1=0,L_1$$
$$\overset{(1)}{V}_1=\overset{(0)}{V}_3=\overset{(1)}{L}_{22}=0 \quad \text{at} \quad x_2=0,L_2. \tag{27}$$

SOLUTION OF THE STABILITY PROBLEM

As was mentioned before, the stability of viscoelastic composite plates subject to in-plane edge loads could be analyzed starting with eqns (20) and (21) or their FSDT counterparts (i.e. eqns (25) and (26)). However, henceforth we will consider the case of constant, in-plane, biaxial edge loads, i.e.

$$\overset{(0)}{L}_{11}[t] = \overset{(0)}{L}_{11}, \quad \overset{(0)}{L}_{22}[t] = \overset{(0)}{L}_{22} \quad \text{and} \quad \overset{(0)}{L}_{12}[t] = 0.$$

For such a case it is more appropriate to solve the stability problem in the Laplace transformed domain. Therefore, we seek the conditions on the edge loads that yield the instability of the system (represented in the present case by the plate).

Stability analysis using the third-order transverse shear deformation theory (TSDT)

As was noted previously the solution of the equation governing the stability of the plate requires the fulfillment of boundary conditions (27). To this end, the following representation of the displacement field $\overset{(1)}{V}_\alpha[x_\omega, t]$ and $\overset{(0)}{V}_3[x_\omega, t]$ satisfying boundary conditions (27) is postulated:

$$\overset{(1)}{V}_1 = \sum_{m=1}^{r} \sum_{n=1}^{r} A_{mn} \cos[\lambda_m x_1] \sin[\lambda_n x_2] f_{mn}[t]$$

$$\overset{(1)}{V}_2 = \sum_{m=1}^{r} \sum_{n=1}^{r} B_{mn} \sin[\lambda_m x_1] \cos[\lambda_n x_2] f_{mn}[t]$$

$$\overset{(0)}{V}_3 = \sum_{m=1}^{r} \sum_{n=1}^{r} C_{mn} \sin[\lambda_m x_1] \sin[\lambda_n x_2] f_{mn}[t] \tag{28}$$

where $\lambda_m = m\pi/L_1$, $\lambda_n = n\pi/L_2$ and A_{mn}, B_{mn}, C_{mn} are constants representing the amplitudes of the displacement quantities. Now the LT of eqns (20) and (21) are eqns (18) and (19), respectively. Thus, introducing the LT of eqns (28) into eqn (18a) corresponding to the free index $\alpha = 1$ yields the following equation:

$$\sum_{m=1}^{r} \sum_{n=1}^{r} (\bar{Y}_{mn}[s] \bar{f}_{mn} + \bar{I}_{mn}[s]) \cos[\lambda_m x_1] \sin[\lambda_n x_2] = 0 \tag{29a}$$

where

$$\begin{aligned} \bar{Y}_{mn}[s] = & -C_{mn}(\bar{\bar{E}}^*_{1111}\lambda_m^3 + \bar{\bar{E}}^*_{1122}\lambda_m\lambda_n^2 + 2\bar{\bar{E}}^*_{1212}\lambda_m\lambda_n^2) + 4A_{mn}(\bar{\bar{E}}^*_{1111}\lambda_m^2 + \bar{\bar{E}}^*_{1212}\lambda_n^2) \\ & + 4B_{mn}(\bar{\bar{E}}^*_{1122}\lambda_m\lambda_n + \bar{\bar{E}}^*_{1212}\lambda_m\lambda_n) - A_{mn}(4\delta_A \bar{\bar{E}}^*_{1111}\lambda_m^2) - B_{mn}(4\delta_A \bar{\bar{E}}^*_{1122}\lambda_m\lambda_n) \\ & - C_{mn}(4\delta_A \bar{\bar{E}}^*_{1111}\lambda_m^3 + 4\delta_A \bar{\bar{E}}^*_{1122}\lambda_m\lambda_n^2) - C_{mn}(5\delta_A\delta_B\rho \bar{\bar{E}}^*_{1133}\lambda_m)s^2 \\ & + A_{mn}\left(\frac{40}{h^2}\bar{E}^*_{1313}\right) + C_{mn}\left(\frac{40}{h^2}\lambda_m \bar{E}^*_{1313}\right) + A_{mn}(4\delta_C\rho)s^2 - C_{mn}(\delta_C\rho\lambda_m)s^2 \end{aligned} \tag{29b}$$

and

$$\bar{I}_{mn}[s] = \{C_{mn}(-5\delta_A\delta_B\rho\bar{\bar{E}}^*_{1133}\lambda_m - \delta_C\rho\lambda_m) + A_{mn}(4\delta_C\rho)\}\{s f_{mn}[0] + \dot{f}_{mn}[0]\}. \tag{29c}$$

The equation corresponding to the free index $\alpha = 2$ can be obtained from eqns (29) by replacing the index 1 with 2, λ_m with λ_n and A_{mn} with B_{mn} (and vice versa). Examining eqns (29) (and their counterparts for the index $\alpha = 2$), we may infer that due to the orthogonality of the sine and cosine functions, we have the following result:

$$\bar{Y}_{mn}[s]\bar{f}_{mn}[s] + \bar{I}_{mn}[s] = 0. \tag{30}$$

As was noted earlier, the stability of a linear dynamical system (of the type represented by eqn (30)) does not depend on the initial conditions (i.e. $\bar{I}_{mn}[s]$) but is simply determined by the nature of its impulse response, i.e. $\mathcal{L}^{-1}\{(\bar{Y}_{mn}[s])^{-1}\}$. Thus we may write the stability equation as follows:

$$\bar{Y}_{mn}[s] = 0 \text{ (and its counterpart for the index } \alpha = 2). \tag{31}$$

Now introducing eqns (28) into eqn (18a) for the case of uniform biaxial compression (i.e. $\overset{(0)}{L}_{11} = \sigma_{11}h$, $\overset{(0)}{L}_{22} = \sigma_{22}h$, $\overset{(0)}{L}_{12} = 0$) we obtain

$$\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} (\bar{W}_{mn}[s]\bar{f}_{mn}[s] + \bar{J}_{mn}[s]) \sin[\lambda_m x_1] \sin[\lambda_n x_2] = 0 \tag{32}$$

where

$$\begin{aligned} \bar{W}_{mn}[s] = {} & A_{mn}(\tfrac{2}{3}h\bar{E}^{*}_{1313}\lambda_m) + B_{mn}(\tfrac{2}{3}h\bar{E}^{*}_{2323}\lambda_n) + C_{mn}(\tfrac{2}{3}h\bar{E}^{*}_{1313}\lambda_m^2 + \tfrac{2}{3}h\bar{E}^{*}_{2323}\lambda_n^2) \\ & + C_{mn}h[\sigma_{11}\lambda_m^2 + \sigma_{22}\lambda_n^2] + C_{mn}(\delta_D \rho h)s^2 \end{aligned}$$

and

$$\bar{J}_{mn}[s] = C_{mn}(\delta_D \rho h)(s f_{mn}[0] + \dot{f}_{mn}[0]).$$

A similar reasoning as above yields

$$\bar{W}_{mn}[s] = 0. \tag{33}$$

Equations (31) (and their counterparts for the index $\alpha = 2$) together with eqn (33) are the three equations governing the stability behavior. This set of three equations represents a homogeneous system of equations in terms of the unknown amplitudes A_{mn}, B_{mn}, C_{mn} (playing the role of an eigenvector). The use of eqns (31) and (33) in conjunction with eqns (29) and (32), allows one to write this system of homogeneous equations in the form:

$$\begin{bmatrix} Z_{11} & Z_{12} & Z_{13} \\ Z_{21} & Z_{22} & Z_{23} \\ Z_{31} & Z_{32} & Z_{33} \end{bmatrix} \begin{Bmatrix} A_{mn} \\ B_{mn} \\ C_{mn} \end{Bmatrix} = 0 \tag{34}$$

where

$$Z_{11} = \left[\left((4\bar{\bar{E}}^{*}_{1111}\lambda_m^2 + 4\bar{\bar{E}}^{*}_{1212}\lambda_n^2 - 4\delta_A \check{\bar{E}}^{*}_{1111}\lambda_m^2 + \frac{40}{h^2}\bar{E}^{*}_{1313}\right) + (4\delta_C \rho)s^2\right]$$

$$Z_{12} = [(4\bar{\bar{E}}^{*}_{1122}\lambda_m\lambda_n + 4\bar{\bar{E}}^{*}_{1212}\lambda_m\lambda_n - 4\delta_A \check{\bar{E}}^{*}_{1122}\lambda_m\lambda_n)]$$

$$\begin{aligned} Z_{13} = {} & \left[\left(-\bar{\bar{E}}^{*}_{1111}\lambda_m^3 - \bar{\bar{E}}^{*}_{1122}\lambda_m\lambda_n^2 - 2\bar{\bar{E}}^{*}_{1212}\lambda_m\lambda_n^2 - 4\delta_A \check{\bar{E}}^{*}_{1111}\lambda_m^3 \right.\right. \\ & \left.\left. - 4\delta_A \check{\bar{E}}^{*}_{1122}\lambda_m\lambda_n^2 + \frac{40}{h^2}\lambda_m \bar{E}^{*}_{1313}\right) - (5\delta_A\delta_B \rho \check{\bar{E}}^{*}_{1133}\lambda_m + \delta_C \rho \lambda_m)s^2\right] \end{aligned}$$

$$Z_{21} = [(4\bar{\bar{E}}^{*}_{1122}\lambda_m\lambda_n + 4\bar{\bar{E}}^{*}_{1212}\lambda_m\lambda_n - 4\delta_A \check{\bar{E}}^{*}_{2211}\lambda_m\lambda_n)]$$

$$Z_{22} = \left[\left(4\bar{\bar{E}}^{*}_{2222}\lambda_n^2 + 4\bar{\bar{E}}^{*}_{1212}\lambda_m^2 - 4\delta_A \check{\bar{E}}^{*}_{2222}\lambda_n^2 + \frac{40}{h^2}\bar{E}^{*}_{2323}\right) + (4\delta_C \rho)s^2\right]$$

$$\begin{aligned} Z_{23} = {} & \left[\left(-\bar{\bar{E}}^{*}_{2222}\lambda_n^3 - \bar{\bar{E}}^{*}_{1122}\lambda_m^2\lambda_n - 2\bar{\bar{E}}^{*}_{1212}\lambda_m^2\lambda_n - 4\delta_A \check{\bar{E}}^{*}_{2222}\lambda_n^3 \right.\right. \\ & \left.\left. - 4\delta_A \check{\bar{E}}^{*}_{2211}\lambda_m^2\lambda_n + \frac{40}{h^2}\lambda_n \bar{E}^{*}_{2323}\right) - (5\delta_A\delta_B \rho \check{\bar{E}}^{*}_{2233}\lambda_n + \delta_C \rho \lambda_n)s^2\right] \end{aligned}$$

$$Z_{31} = [(\tfrac{2}{3}h\bar{E}^{*}_{1313}\lambda_m)]$$

$$Z_{32} = [(\tfrac{2}{3}h\bar{E}^{*}_{2323}\lambda_n)]$$

$$Z_{33} = [(\tfrac{2}{3}h\bar{E}^{*}_{1313}\lambda_m^2 + \tfrac{2}{3}h\bar{E}^{*}_{2323}\lambda_n^2 + h[\underline{\sigma}_{11}\lambda_m^2 + \underline{\sigma}_{22}\lambda_n^2]) + (\delta_{\rm D}\rho h)s^2].$$

From eqn (34) it is seen that for non-trivial solutions of A_{mn}, B_{mn}, C_{mn}, the following determinantal equation is to be fulfilled

$$\det\,[Z_{ij}] = 0. \tag{35}$$

Equation (35) yields a characteristic equation of the form

$$\frac{P_{mn}[s]}{Q_{mn}[s]} = 0 \tag{36}$$

where $P_{mn}[s]$ and $Q_{mn}[s]$ are polynomials in s. Thus, the zeros of eqn (36) are determined from

$$P_{mn}[s] = 0. \tag{37}$$

Equation (37) is the characteristic equation of the system (represented by the plate subjected to uniform biaxial compression). The zeros of this equation, i.e. the roots s_i of $P_{mn}[s]$, are the eigenvalues of the system which in general are complex quantities. They decide the nature of $f_{mn}[t]$ and hence the stability of the system. When $\mathrm{Re}\,[s_i] > 0$, $f_{mn}[t]$ becomes unbounded with time and due to the nature of s_i the following cases of instability may arise. (i) $\mathrm{Im}\,[s_i] = 0$: in this case $f_{mn}[t]$ grows exponentially with time, and we have instability by divergence. (ii) $\mathrm{Im}\,[s_i] \neq 0$: in this case $f_{mn}[t]$ has an oscillatory growth with time. This leads to instability by flutter.

Therefore, the stability problem is reduced to the examination of the nature of the zeros of the characteristic equation, eqn (37). The coefficients of the characteristic polynomial, $P_{mn}[s]$, in eqn (37) can be varied by suitably varying the in-plane edge loads $\underline{\sigma}_{11}$ and $\underline{\sigma}_{22}$ in order to yield the instability boundaries of the system.

Stability analysis using a first-order transverse shear deformation theory (FSDT)

An analogous procedure to that developed for the TSDT applied to eqns (25) and (26), yields the characteristic equation of the system in exactly the same form as given by eqn (37), but with different coefficients. For this case the coefficients are

$$Z_{11} = \left[\bar{\bar{E}}^{*}_{1111}\left(\frac{h^3}{12}\lambda_m^2\right) + \bar{\bar{E}}^{*}_{1212}\left(\frac{h^3}{12}\lambda_n^2\right) + \bar{E}^{*}_{1313}(K^2h) + s^2\delta_{\rm C}m_1\right]$$

$$Z_{12} = \left[\bar{\bar{E}}^{*}_{1122}\left(\frac{h^3}{12}\lambda_m\lambda_n\right) + \bar{\bar{E}}^{*}_{1212}\left(\frac{h^3}{12}\lambda_m\lambda_n\right)\right]$$

$$Z_{13} = [\bar{E}^{*}_{1313}(K^2h\lambda_m)]$$

$$Z_{21} = Z_{12}$$

$$Z_{22} = \left[\bar{\bar{E}}^{*}_{2222}\left(\frac{h^3}{12}\lambda_n^2\right) + \bar{\bar{E}}^{*}_{1212}\left(\frac{h^3}{12}\lambda_m^2\right) + \bar{E}^{*}_{2323}(K^2h) + s^2\delta_{\rm C}m_1\right]$$

$$Z_{23} = [\bar{E}^{*}_{2323}(K^2h\lambda_n)]$$

$$Z_{31} = Z_{13}$$

$$Z_{32} = Z_{23}$$

$$Z_{33} = [\bar{E}^{*}_{1313}(K^2h\lambda_m^2) + \bar{E}^{*}_{2323}(K^2h\lambda_n^2) + h(\underline{\sigma}_{11}\lambda_m^2 + \underline{\sigma}_{22}\lambda_n^2) + (\delta_{\rm D}m_0)s^2]. \tag{38}$$

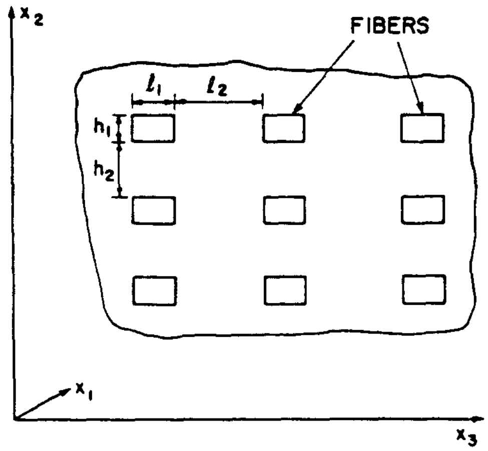

Fig. 1. Arrangement of fibers in matrix.

By paralleling the procedure presented previously for the TSDT we may obtain within FSDT the stability boundaries for viscoelastic orthotropic plates subjected to uniform in-plane edge loads.

MATERIAL PROPERTY DETERMINATION

The constitutive law of a three-dimensional anisotropic viscoelastic body (see eqn (6)) will be fully determined by expressing explicitly the relaxation moduli as functions of time. This task will be accomplished by using a micromechanical model. Such a model, developed in Aboudi (1984, 1986, 1987) will allow one to predict the overall behavior (i.e. effective properties E_{ijmn}) of the unidirectional fiber-reinforced composite, in terms of the properties of its constituents (i.e. fiber and matrix).

This model assumes that continuous fibers extend in the x_1-direction and are arranged in a doubly periodic array in the x_2- and x_3-directions (Fig. 1). The cross-section of the rectangular fibers is $h_1 l_1$, while h_2, l_2 represent their spacing in the matrix. Due to this periodic arrangement, we need to analyze only a representative element as shown in Fig. 2. This representative cell contains four sub-cells identified by β, $\gamma = 1, 2$. Four local coordinate systems defined by x_1, $\bar{x}_2^{(\beta)}$, $\bar{x}_3^{(\gamma)}$, and having their origins at the center of each sub-cell, are displayed in Fig. 2. The following first-order displacement expansion in each sub-cell is considered:

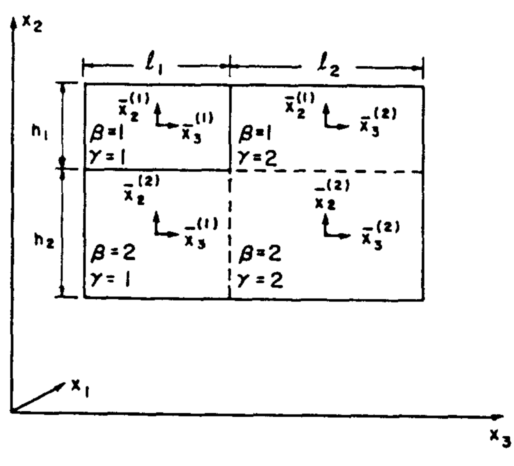

Fig. 2. Representative volume element for a fiber-matrix composite.

$$V_i^{(\beta\gamma)} = W_i^{(\beta\gamma)} + \bar{x}_2^{(\beta)} \phi_i^{(\beta\gamma)} + \bar{x}_3^{(\gamma)} \psi_i^{(\beta\gamma)} \tag{39}$$

where $W_i^{(\beta\gamma)}$ are the displacement components of the center of each sub-cell with $\phi_i^{(\beta\gamma)}$, $\psi_i^{(\beta\gamma)}$ characterizing the linear dependence of the displacements on the local coordinates $\bar{x}_2^{(\beta)}$, $\bar{x}_3^{(\gamma)}$. Here the repeated Greek indices (in parentheses) do not imply summation.

By using the relations of continuity of displacements and of tractions at the interfaces between sub-cells (see Aboudi (1986) for details), we can obtain a system of linear equations in terms of the microvariables $\phi_i^{(\beta\gamma)}$, $\psi_i^{(\beta\gamma)}$. Solving for the microvariables we can obtain the explicit constitutive law relating average stresses to average strains (Aboudi, 1986). The detailed expressions for the resulting constitutive law of the elastic continuum can be found in Aboudi (1987). Their employment along with the elastic–viscoelastic correspondence principle yields the relevant micromechanical equations pertaining to a viscoelastic fiber-reinforced composite (Chandiramani, 1987). These equations relate the Carson transformed effective moduli of the composite (i.e. $\bar{E}^*_{ijmn}$), to the Carson transformed effective moduli of the constituents (i.e. the fiber and matrix). The model developed in Aboudi (1987) considers the constituents as being transversely isotropic. Furthermore, both the fiber and the matrix may be modelled by a linearly viscoelastic constitutive law. However, for the purpose of the present work, the fiber was modelled as a linearly elastic isotropic material, while the matrix as a linearly viscoelastic isotropic one.

The properties of the isotropic, elastic, boron fibers considered in Aboudi and Weitsman (1974) are as follows:

$$G^{(f)} = 25 \times 10^6 \text{ psi}$$
$$K^{(f)} = 33.2 \times 10^6 \text{ psi}$$

where $G^{(f)}$, $K^{(f)}$ represent the shear and bulk moduli, respectively. Using the above properties, we obtain

$$\lambda^{(f)} = K^{(f)} - \tfrac{2}{3}G^{(f)} = 16.53 \times 10^6 \text{ psi}$$

$$E^{(f)} = \frac{G^{(f)}(3\lambda^{(f)} + 2G^{(f)})}{\lambda^{(f)} + G^{(f)}} = 6.426 \times 10^7 \text{ psi}$$

$$\mu^{(f)} = \frac{(K^{(f)} - \frac{2}{3}G^{(f)})}{2(K^{(f)} + \frac{1}{3}G^{(f)})} = 0.1990. \tag{40}$$

Using the properties for the isotropic viscoelastic epoxy matrix considered by Mohlenpah *et al.* (1969) and Schapery (1972) we can represent them as a three-parameter solid in the following manner:

$$E^{(m)}[t] = 0.8 \times 10^5 + 0.18 \times 10^6 \text{ e}^{-0.4115 \times 10^{-3} t} \text{ for } 0 \leqslant t \leqslant 2000 \text{ h}$$

$$\mu^{(m)}[t] = 0.372 - 0.007 \text{ e}^{-0.2403 \times 10^{-2} t} \text{ for } 0 \leqslant t \leqslant 2000 \text{ h} \tag{41}$$

where the time t is in min.

Now in order to obtain the time-dependent relaxation moduli, $E_{ijmn}[t]$, for the viscoelastic orthotropic composite plate, we must invert the corresponding Laplace transformed moduli given by $(s)^{-1}\bar{E}^*_{ijmn}[s]$.

A rigorous treatment of the problem of LT inversion is given by Bellman and Kalaba (1966) and this method, which was chosen for this problem, has been effectively used in Swanson (1980) for dynamic viscoelastic problems. Within this method, referred to as Bellman's technique, the definition of the LT is used to invert the LT by means of a Gaussian quadrature using orthogonal polynomials. Due to their excellent convergence properties, Legendre polynomials were mainly used in Bellman and Kalaba (1966).

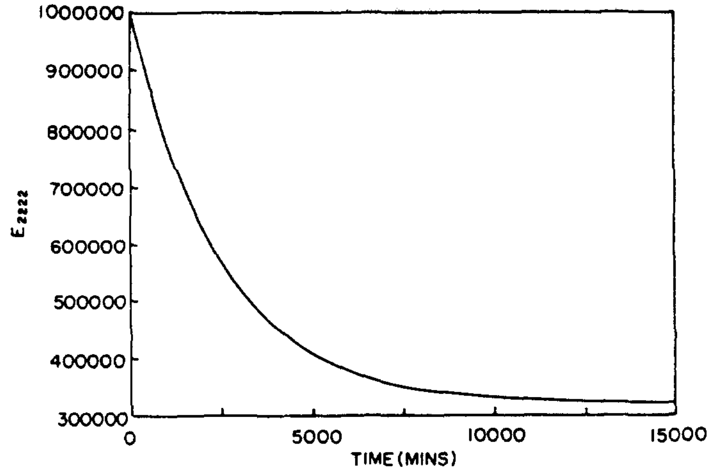

Fig. 3. Material property for the orthotropic plate: $E_{2222} = E_{3333}$.

It was observed that for the order of the polynomial $N = 10$, the convergence has been attained to provide accurate results. For $N = 15$, the results were almost the same as for $N = 10$.

The results obtained for $E_{ijmn}[t]$ vs time $[t]$ are shown in Figs 3–7 for the case of equally-spaced square fibers (in which case we obtain six independent material constants for an orthotropic body instead of nine (Aboudi, 1987)). When these plots are fitted by an exponential series corresponding to a three-parameter solid, we obtain the following results:

$$
\begin{aligned}
E_{1111} &= 0.2903 \times 10^{8} + 0.2500 \times 10^{6}\, e^{-(0.3746 \times 10^{-3})t} \\
E_{2222} = E_{3333} &= 0.3212 \times 10^{6} + 0.6769 \times 10^{6}\, e^{-(0.3986 \times 10^{-3})t} \\
E_{1122} = E_{1133} &= 0.1294 \times 10^{6} + 0.2633 \times 10^{6}\, e^{-(0.3785 \times 10^{-3})t} \\
E_{2233} &= 0.1304 \times 10^{6} + 0.2609 \times 10^{6}\, e^{-(0.3687 \times 10^{-4})t} \\
E_{1212} = E_{1313} &= 0.6921 \times 10^{5} + 0.1548 \times 10^{6}\, e^{-(0.4356 \times 10^{-3})t} \\
E_{2323} &= 0.5321 \times 10^{5} + 0.1194 \times 10^{6}\, e^{-(0.4365 \times 10^{-3})t}.
\end{aligned}
\tag{42}
$$

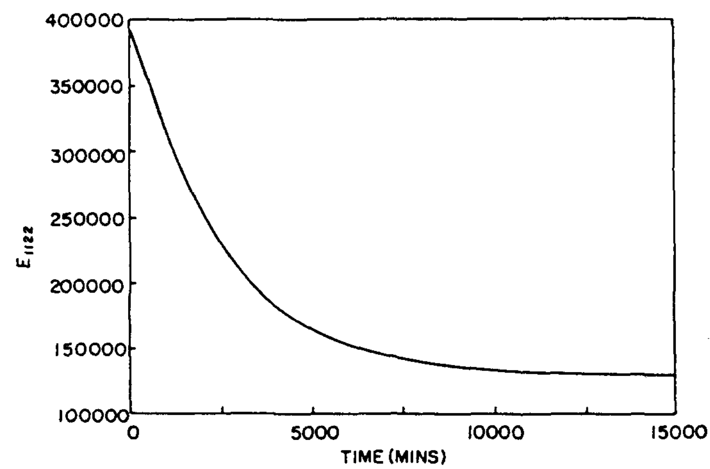

Fig. 4. Material property for the orthotropic plate: $E_{1122} = E_{1133}$.

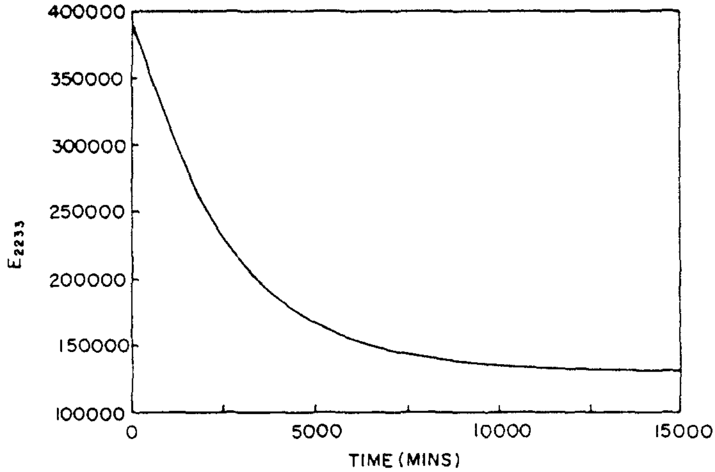

Fig. 5. Material property for the orthotropic plate: E_{2233}.

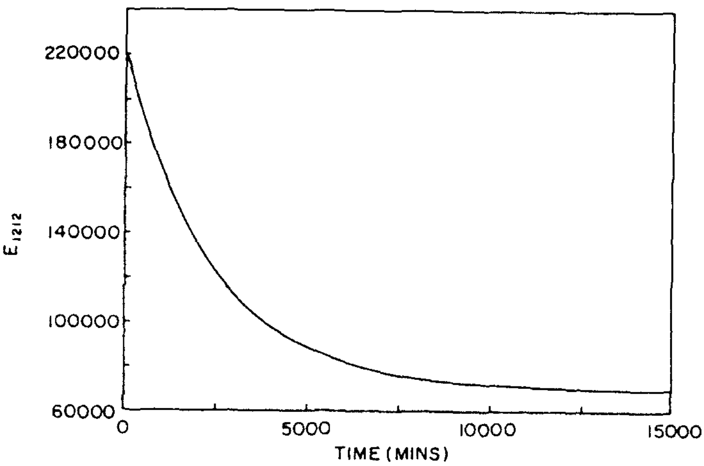

Fig. 6. Material property for the orthotropic plate: $E_{1212} = E_{1313}$.

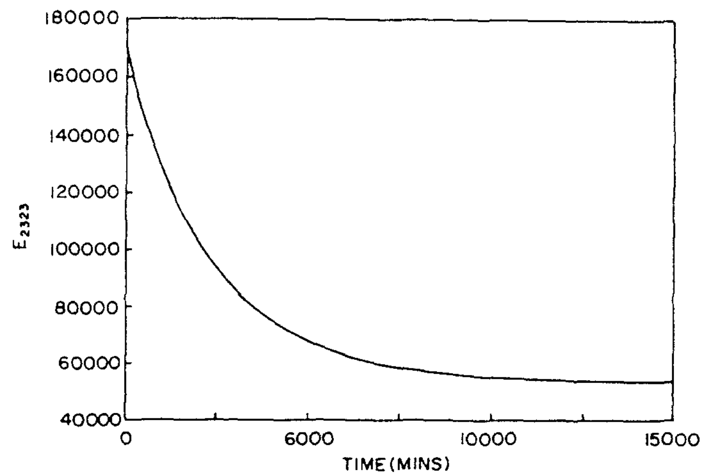

Fig. 7. Material property for the orthotropic plate: E_{2323}.

NUMERICAL RESULTS AND CONCLUSIONS

Numerical results

The stability boundary was obtained by solving the characteristic polynomials associated with TSDT (eqn (37)), and its FSDT counterpart. This was done by using the IMSL subroutine ZPOLR. The numerical applications were considered for an orthotropic,

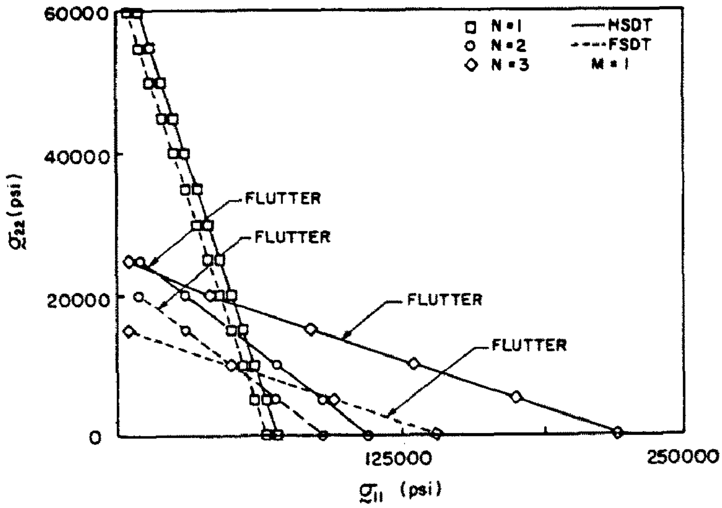

Fig. 8. Stability boundary for the orthotropic viscoelastic plate $L_1/h = 4.8$; biaxial compression; $\delta_\Lambda = 1$.

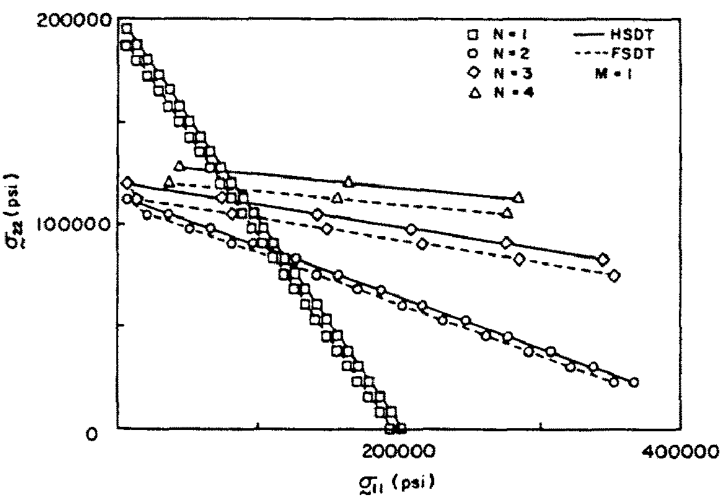

Fig. 9. Stability boundary for the orthotropic elastic plate; $L_1/h = 4.8$; biaxial compression; $\delta_\Lambda = 1$ or $\delta_\Lambda = 0$.

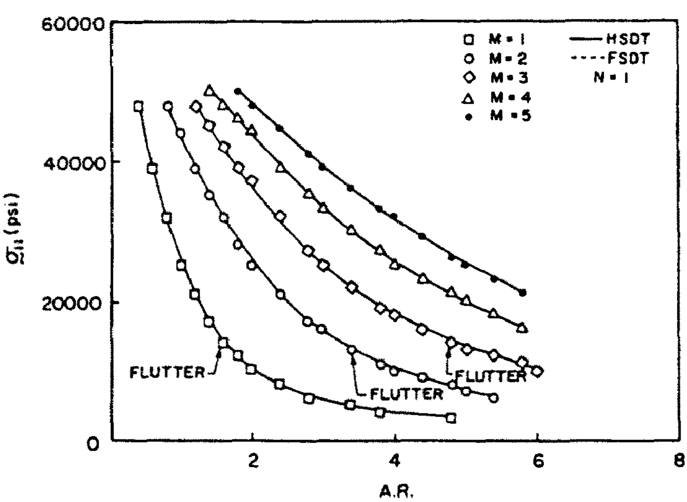

Fig. 10. Stability boundary for the orthotropic viscoelastic plate; $L_1/h = 24$; uniaxial compression; $\delta_\Lambda = 1$ or $\delta_\Lambda = 0$.

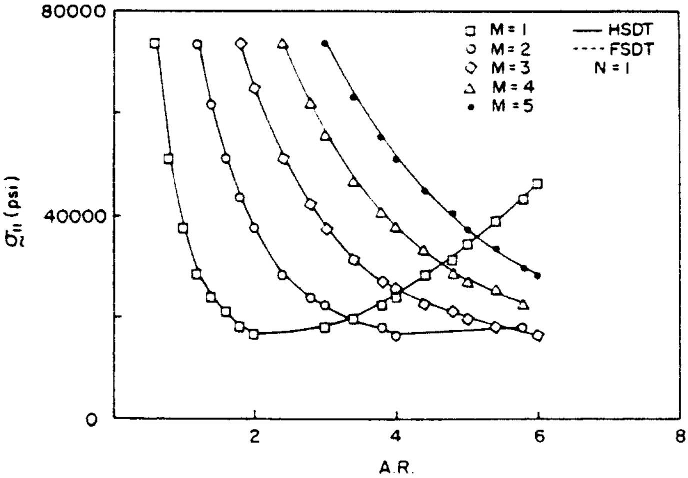

Fig. 11. Stability boundary for the orthotropic elastic plate; $L_1/h = 24$; uniaxial compression; $\delta_A = 1$ or $\delta_A = 0$.

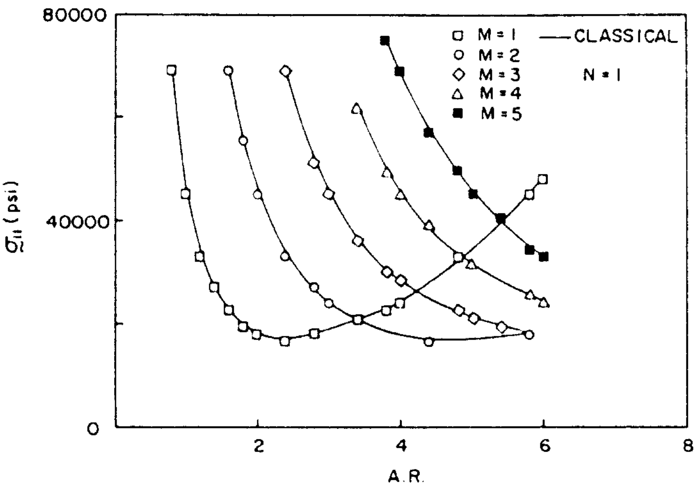

Fig. 12. Comparison of stability boundaries for the orthotropic elastic plate; $L_1/h = 24$; uniaxial compression.

viscoelastic plate. By invoking the initial value theorem for the Laplace transformed material properties appearing in eqns (22) the numerical applications could incorporate also their elastic counterparts.

All cases above were considered so as to obtain an "exact" dynamic solution, i.e. for $\delta_A = \delta_B = \delta_C = \delta_D = 1$ where δ_B, δ_C, δ_D are tracers identifying the dynamic effect in σ_{33}, rotary inertia and transverse inertia, respectively, while δ_A is a tracer identifying the overall (i.e. static and dynamic) effect of σ_{33}. It was observed that the inclusion or exclusion of the inertia terms does not affect the results.

The results associated with the classical Kirchhoff plate theory may be obtained as a special case of the FSDT by considering $K^2 \to \infty$ which is equivalent to considering infinite transverse shear rigidities. The results obtained in this study are not universal since a non-dimensional analysis was not possible due to the inherent complexity of the problem.

The stability boundaries are shown in Figs 8–12 for the following cases:

(i) viscoelastic, flat plate;
(ii) elastic, flat plate.

Cases (i) and (ii) are considered for thick ($L_1/h = 4.8$) as well as thin plates ($L_1/h = 24$). In addition, the following sub-cases were considered. (a) *Biaxial compression* for which case, the aspect ratio (A.R. $\equiv L_1/L_2$) of the plate was taken as unity. The values of the in-plane normal edge loads σ_{11} vs σ_{22} are plotted to obtain the stability boundaries. (b) *Uniaxial compression*: within this case, the aspect ratio, A.R., was varied and the corresponding value of g_{11} was plotted in order to obtain the stability boundaries. For all plots shown, M and N denote the mode numbers in the x_1- and x_2-directions, respectively. It was observed that for biaxial compression, the stability boundaries corresponding to $M = 1$ were the lowest ones, whereas for uniaxial compression, those corresponding to $N = 1$ were the lowest ones. Therefore, in each of these two sub-cases, only the lowest stability boundaries were displayed. For all the cases, unless otherwise indicated, instability occurs by divergence only. Flutter boundaries are indicated on the figures. For the uniaxial compression case, flutter instability occurs to the right of the arrow appearing in the figures.

Conclusions

In this paper, a stability analysis of orthotropic, viscoelastic rectangular plates has been accomplished. The equations governing the stability were derived by using the correspondence principle technique. The material properties were obtained by considering the micromechanical model developed in Aboudi (1987). In the modeling of the problem, the Boltzmann hereditary constitutive law for a three-dimensional viscoelastic medium has been used. The stability problem was analyzed in the Laplace transformed space in order to determine the asymptotic stability behavior.

The special cases considered in the numerical applications allow one to conclude the following.

(1) The stability boundary determined for a viscoelastic plate is lower (i.e. more critical) than its elastic counterpart.

(2) Figures 8 and 9 reveal that σ_{33} may influence the viscoelastic stability boundary in a strong and beneficial way. However, as may be concluded from Figs 10 and 11, for thin panels the influence of σ_{33} on the instability boundaries becomes insignificant.

(3) Figures 8 and 9 as well as Figs 10 and 11 reveal that transverse shear deformation effects are more pronounced in viscoelastic plates than in their elastic counterparts. However, this feature is much more accentuated in the case of thick panels (Figs 8 and 9) than in the case of thin ones (Figs 10–12).

(4) The analysis performed here allows one to obtain the nature of loss of stability, i.e. either by divergence or by flutter. It was revealed in Figs 8 and 10 that for an orthotropic viscoelastic plate, the instability may result both by divergence and by flutter.

(5) It is observed that for large aspect ratios (L_1/L_2) the stability boundaries come closer to their counterparts determined within the classical Kirchhoff theory of plates.

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